M.Sc. - Chemistry
I Semester End Examination - May 2022
Mathematics for Chemists (Soft Core)

Course Code: MCH105S **QP Code: 11011**
Time: 3 hours **Total Marks: 70**

Instruction: Answer Question No. 1 and any FIVE of the remaining.

1. Answer any *TEN* questions **(2×10=20)**

a) Find the unit vector perpendicular to the vectors $\vec{a} = 3\hat{\imath} + \hat{\jmath} - \hat{k}$,
$\vec{b} = \hat{\imath} - \hat{\jmath} + 2\hat{k}$ *and* $\vec{c} = -\hat{\imath} + 2\hat{\jmath} + 4\hat{k}$.

b) Find the sine of the angle between $\vec{a} = 2\hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{b} = 3\hat{\imath} + \hat{\jmath} + \hat{k}$

c) If A = $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 2 & -9 \end{bmatrix}$ then find trace of A.

d) Write the characteristic equation for A=$\begin{bmatrix} 2 & 5 \\ 4 & 5 \end{bmatrix}$

e) Find the n^{th} derivative of $y = e^{ax} cosbx$.

f) Show that f(x, y) = $x^3 + y^3 - 3xy + 1$ is minimum at the point (1,1).

g) Solve sec^2x tany dx + sec^2y tanx dy = 0.

h) Two dice are thrown simultaneously. Find the Sample space

i) Evaluate $\int xsinxdx$

j) Solve the differential equation $y' = e^{3x-2y}$

k) Solve the differential equation $y' = e^{3x-2y}$

l) Find $\frac{dy}{dx}$ if $x=at^2$ and y=2at.

2. a) Find the ratios in which P divides AB where A=(3,2,-4) , B=(9,8,-10) and P= (5,4,-6).
b) Find the volume of the tetrahedron whose vertices are given by (3, 2, 1), (1, 2, 4), (4, 0, 3) and (1,1,7).

(5+5=10)

3. a) Prove that $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ = (a-b) (b-c) (c-a).

b) Find the eigenvalues and eigenvectors for the matrix A= $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$

(5+5=10)

4. a) Find the inverse of the matrix

$$A=\begin{bmatrix}1 & 2 & 3\\ 1 & 3 & 3\\ 2 & 4 & 3\end{bmatrix}$$

b) Solve :

$$x + y + z = 3$$
$$3x + 4y + 7z = 14$$
$$x - y + z = 1$$

by Cramer's rule.

(5+5=10)

5. a) If $y = (\sin^{-1} x)^2$. Show that $(1-x^2)y_{n+2} - (2n+1)y_{n+1} - n^2 y_n = 0$

b) If $u = \frac{1}{\sqrt{x^2+y^2+z^2}}$ then show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$

(5+5=10)

6. a) A spherical balloon is inflated at the rate of 35 cc/s. Find the rate at which the surface area of the balloon increases when its diameter is 14cm.

b) Find the equation of the tangent and normal to the curve $y^2 = \frac{x^3}{2a-x}$ at (a,a).

(5+5=10)

7. a) Solve *i)* $\int \frac{8+3t}{10t^2+13t-} dt$

ii) Find the area bounded by the cure $x = a\cos\theta$ and $y = b\sin\theta$, $0 \le \theta \le 2\pi$

b) Solve $(D^3 - 4D^2 + 4D)y = 0$ **(5+5=10)**

8. a) Fit a straight line for the following data using least squares method:

x	1	2	3	4	5
y	5	3	2	1	3

b) Find the Fourier series of the function $f(x) = x,\ -\pi < x < \pi$ **(4+6=10)**

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